MONETARY EQUILIBRIUM IN OLG WITH HABIT PERSISTENCE

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Abstract

This paper introduces the money into OLG framework with assuming habit formation. We show that still there exists optimal stationary monetary equilibrium which is above the equilibrium without habit persistence. Besides the theoretical results, we also analyse this issue by assuming that agents have logarithmic preferences with habit persistence.

Key Words and Phrases: Money, OLG, Habit Persistence

1 Introduction

The last two decades have seen a renewal of the popularity in the old idea that habit persistence hypothesis may serve a tool in solving puzzles concerning consumption behaviour (excess smoothness and sensitivity), stock market behaviour (equity premium puzzle) as well as concerning with the relationship between growth and savings. The high level of interest on this issue has been motivated by the empirical findings that have not been successful in explaining with using traditional time-separable utility function. Instead, several authors have argued that non-separable utility in preferences may be introduced to explain the puzzles in modern macroeconomic theory. For instance, Kydland and Prescott (1982), Eichenbaum et.al. (1988) and Kennan (1988) among others incorporate inter-temporally non-separable utility function to investigate the observed behaviour of consumption and labor during the business cycle.. Moreover, Alessie and Lusardi (1997) consider the models of habit formation and derive closed-form solutions for consumption and saving under certainty equivalence and uncertainty. They conclude that consumption depends not only permanent income but also on past consumption. Seckin (2001) show that habit-forming consumption and leisure do move in opposite directions which is consistent with the observed procyclicy of aggregate hours worked. On the other hand, Fuhrer (2000) examines the habit consumption specification in a monetary-policy model and concludes that responses of both spending and inflation to monetary-policy actions are improved by this specification.. Faria (2001) considers the hypothesis of habitual consumption in the Sidrouski model, and observes that the demand for money in his model is greater than the Sidrauski model and nominal interest rates rather than the Sidrauski model.

In conclusion, many of either empirical or theoretical studies emphasize on solving of the puzzles which are popular in the theory of macroeconomics in recent years. However, the interesting analysis on habit consumption may be made in the overlapping generates (OLG) framework because habits have been widely regarded as the consumption of commodities such as cigarettes and alcohol and consumers, when they are young, are usually learning these habits which lead to reduce their future utility. In other words, OLG set up provides the designation that the agent in period t starts to become familiar with the habits and only is affected in the period of t+1. Moreover, this also causes to occur heterogeneity in the preferences between the periods t and t+1.

There is one attempt in analyzing habit persistence in OLG context performed by Lahiri and Puhakka (1998). This paper extends their work by introducing money to the OLG framework and even distinguishes itself by proving that there exists optimal stationary monetary equilibria under the habit formation. Moreover, we solve the consumer optimization problem by assuming the logarithmic preferences in which there is not habitual consumption in period t but is at the period of t+1.

Section 2 gives the economic environment and the existence of monetary equilibrium with habit persistence. Section 3 shows the solution of the consumer optimization problem under the assumption of logarithmic preferences, and Section 4 includes the conclusion. Appendix is reserved for proofs.

2 The Environment and Monetary Equilibrium

Consider an overlapping generations economy with one agent born at each t$\geqslant$1 and an initial old person at t=1. Agents live two periods. Young agents born at ate t have strictly positive endowment pattern (y_t,y_{t+1}) which are non-storable consumption goods. At any date t, the population consists of N_t young agents and N_{t-1} old agents where N_t=$nN_{t-1}$with n>0. Preferences with habit of a young agent given by the utility function $u(c_t, \widetilde{c}_{t+1})$ which is twice differentiable smooth function. Assume that the two goods in the utility function are normal goods and marginal rate of substitution (MRS) function is

$$\Lambda(c_t, \widetilde{c}_{t+1}) = \frac{u_1(c_t, \widetilde{c}_{t+1})}{u_2(c_t, \widetilde{c}_{t+1})} \tag{1}$$

where $\widetilde{c}_{t+1}$refers to the habitual consumption of the old agent in the subtractive form such that $\widetilde{c}_{t+1} = c_{t+1} - \gamma c_t$ with the habit parameter $\gamma \in (0, 1)$.Notice that subscripts, 1 and 2, for the partial derivatives with respect to the arguments, respectively, in the parenthesis..

Now suppose that M_t be the stock of fiat money which evolves according to

$M_t = \tau M_{t-1}$ where τ is a positive constant money growth rate. Assume that new money is given equally to the old generation as a lump-sum transfer payment (or tax) at time t, so is $TR_t = (\tau - 1)M_t$. The budget constraints of a young agent born in period t are given

$$c_t = y_t - \frac{m_t}{P_t} \tag{2}$$

$$c_{t+1} = y_{t+1} - \frac{m_t}{P_{t+1}} + TR_t \tag{3}$$

where $m_t = \frac{M_t}{N_t}$ and P_t is the price level. Let $\widehat{m}_t$be the real money holdings in the equilibrium such that $\widehat{m}_t = \frac{m_t}{P_t}$. Correspondinglyndingly, budget constrains (2) and (3) become $c_t = y_t - \widehat{m}_t$ and $c_{t+1} = y_{t+1} - n\widehat{m}_{t+1}$, respectively, in the equilibrium.

Now consider agents who live for two periods maximize the present value of lifetime utility with habit given by

$$U^* = u(c_t) + \beta u(\widetilde{c}_{t+1}) \tag{4}$$

and first order conditions gives MRS as follows

$$\Lambda = \frac{u_1(c_t, c_{t+1}) - \gamma\beta u_2(c_t, c_{t+1})}{\beta u_2(c_t, c_{t+1})} \tag{5}$$

$$\Lambda = \frac{u_1(c_t, c_{t+1})}{\beta u_2(c_t, c_{t+1})} - \gamma \tag{6}$$

Since an old agent receives transfer payments which is a function of nonstochastic money supply, then MRS in monetary equilibrium without habit as shown on the right hand side of equation (5) is obtained with some simple manipulations,

$$\frac{u_1(c_t, c_{t+1})}{\beta u_2(c_t, c_{t+1})} = \frac{P_t}{P_{t+1}} = \frac{n}{\tau} . \frac{\widehat{m}_{t+1}}{\widehat{m}_t} \tag{7}$$

By considering the budget constraint, the inequalities $c_t < y_t$ and $c_{t+1} > y_{t+1}$ are valid in monetary equilibrium, and, furthermore, under the that goods are normal, so we may set,

$$\frac{\widehat{m}_{t+1}}{\widehat{m}_t} > (\Lambda + \gamma)\frac{\tau}{n} \tag{8}$$

for all t$\geq$ 1.Let $\alpha = (\Lambda + \gamma)\frac{\tau}{n}$ be denoted for simplifying the notation through the rest of the paper.

Proposition 1 $\alpha < 1$ *is necessary and sufficient condition for the existence of at least one monetary equilibrium when the old agent has habit persistence.*

Proof. *By the contrary, suppose that* $\alpha \geq 1$ *and, hence, the inequality (8) be* $\frac{\widehat{m}_{t+1}}{\widehat{m}_t} > \alpha \geq 1$*. First consider the case* $\alpha = 1$ *implying that* $\widehat{m}_{t+1} > \widehat{m}_t$*. Let* $\Delta_{(\alpha)}$ *be the constant depending upon* α *such that* $\Delta_{(\alpha)} > 1$*. Choose* $\Delta_{(\alpha)}$ *satisfying* $\widehat{m}_{t+1} = \Delta_{(\alpha)}.\widehat{m}_t$*.*

Notice that this set up generates a sequence such that $\widehat{m}_{t+1} > \widehat{m}_t$. Obviously, this is monotonically increasing sequence an is not bounded so that $\lim_{t\to\infty}(\widehat{m}_t) = \infty$. Second, consider that α>1 and let $\widetilde{\Delta}_{(\alpha)}$ be a constant such that $\widetilde{\Delta}_{(\alpha)} > \Delta_{(\alpha)}$. So, pick up $\widetilde{\Delta}_{(\alpha)}$ appropriately that satisfies $\widehat{m}_{t+1} = \widetilde{\Delta}_{(\alpha)}.\widehat{m_t}$ which again gives $\lim_{t\to\infty}(\widehat{m}_t) = \infty$. These results are inconsistent with an equilibrium of real money balances bounded above such that $\widehat{m}_t \leq y_t - c_t$. For the sufficiency, let $\widehat{m}$ be steady state equilibrium of real money balances, so that $\widehat{m} = \widehat{m}_{t+1} = \widehat{m}_t$. From the budget constraint of the young, $\widehat{m} \leq y_t - c_t$ or $\widehat{m} \in (0, c_t - y_t]$. Note that $\widehat{m}$ be any real number in this interval, thus, the existence of $\widehat{m}$ is trivial.

Proposition 2 *If $\tau \leq 1$, than an optimal stationary monetary equilibrium with habit persistence exists.*

Proof. *By following the budget constraints of young and old agents,*

$$c_t + \frac{\tau}{n}c_{t+1} \leq y_t + \frac{\tau}{n}y_{t+1} + (\tau - 1)\widehat{m} \tag{9}$$

where $\widehat{m}$ is the stationary real money balances. Suppose that ($\widehat{c}_1, \widehat{c}_2$) is the consumption allocation that maximizes an agent's utility subject to (9). On the other hand, without money transfers,

$$c_t + \frac{c_{t+1}}{n} \leq y_t + \frac{y_{t+1}}{n} \tag{10}$$

and assume that the bundle (c_1^*, c_2^*) maximizes utility subject to this budget constraint (10). If τ <1, then the budget line with the inclusion of money might be steeper than and above the budget without money. So, by revealed preference, $(\widehat{c}_1, \widehat{c}_2) \succ (c_1^*, c_2^*)$. Obviously, if $\tau \leq$1, then again by revealed preference, $(\widehat{c}_1, \widehat{c}_2) \succeq (c_1^*, c_2^*)$ is valid.

Corollary 3 *Monetary equilibrium with habit persistence is above the equilibrium of without habit persistence in the limit.*

Proof. *By the definition of α and with the Proposition 1,*

$$\alpha = \frac{\tau}{n}\Lambda + \frac{\tau}{n}\gamma < 1 \tag{11}$$

Let $\alpha^* = \frac{\tau}{n}\Lambda$ be the component without habit formation. It's clear that $\alpha^* < 1 - \frac{\tau}{n}\gamma$ when $\gamma \in (0,1)$ and $\tau\gamma < \Pi$. So, $\alpha^* < \alpha$. In the limit, consider the dynamics of the real money balances in the equality that $m_{t+1}^* = \alpha^* m_t^*$ and let m^* be the monetary equilibrium such that $m^* = \lim_{t\to\infty}(m_t^*) = \inf(m_t^*)$ due to the fact that m_t^* is bounded monotonically decreasing sequence. In a similar way, for $\alpha^* < \alpha$, suppose the equality that $\widehat{m}_{t+1} = \alpha\widehat{m}_t$ and again let $\widehat{m} = \underset{t\to\infty}{lim}(\widehat{m}_t) = \inf(\widehat{m}_t)$. Of course, the generated sequences $\widehat{m}_t > m_t^*$ for all $t \in T$ due to the fact that $\alpha > \alpha^*$ which asserts that $\widehat{m} > m^*$. This completes the proof.

3 Habit with Logarithmic Preferences

We use the specification of preferences which first proposed by Abel (1990) and, Fuhrer (2000) and Carroll et. al. (2000) also used this type of utility function in the very recent. The stock of habits are evolved as given by the function:

$$h_{t+1} = (1-\varepsilon)h_t + \varepsilon c_t \tag{12}$$

This is implying that the level of habit is a weighted average of the stream of the past consumption. The parameter (1-ε) measures the persistence of the habit stock (h_t). Note further that if let ε=1, then the habit stock at date t+1 is only the consumption of at t, or, in other sense, today's consumption only affects tomorrow's utility.

We assume that agents have familiar with habit when they start to earn money and consume own money when they are young so that this affects negatively only the consumption when they are old. Accordingly, we set ε=1 an assume that agents have logarithmic preferences with the inclusion of multiplicative habit,

$$(c_t, \widetilde{c}_{t+1}) = \ln c_t + \ln(\frac{c_{t+1}}{h_{t+1}^{\gamma}}) \tag{13}$$

where γ be the habit parameter. It's clear that setting $\gamma = 1$ implies that old agent matters with only relative consumption whereas engages in absolute consumption when $\gamma = 0$. Obviously, taking $\mathrm{h}_{t+1} = c_t$ and substituting into (13) give the preference,

$$u(c_t, \widetilde{c}_{t+1}) = (1 - \gamma) \ln c_t + \ln(c_{t+1}) \tag{14}$$

which indeed indicates that logarithmic preferences leads to make habit effective on the consumption of the agent when she is young. Hence, agents would like to maximize the utility given (14) subject to the budget constraints (2) and (3). Let Lagrangian be as follows:

$$L = (1 - \gamma) \ln c_t + \ln(c_{t+1}) + \lambda \left[y_{t+1} + \frac{\tau P_t}{P_{t+1}} y_t - c_{t+1} - \frac{\tau P_t}{P_{t+1}} c_t \right] \tag{15}$$

First order conditions will give the consumption path:

$$c_{t+1} = \frac{P_t}{P_{t+1}} . \frac{\tau}{1 - \gamma} . c_t \tag{16}$$

and setting money market equilibrium as,

$$y_t - c_t = s_t = \frac{m_t}{P_t} \tag{17}$$

where s_t implies the saving of the young at date t. Plugging the result in (16) into budget constraints and with using the money market equilibrium,

$$\frac{m_t}{P_t} = \frac{1}{2 - \gamma} \left\{ y_t - \frac{1 - \gamma}{\tau} y_{t+1} \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{18}$$

So, this assures that $(\frac{m_t}{P_t})_{\gamma\in(0,1)} > (\frac{m_t}{P_t})_{\gamma=0}$ and is consistent with our main result which is proposed and proved in the corollary.

4 Conclusion

Fiat money is introduced into OLG framework with habit persistence. Micro-based issues are analyzed and show that monetary equilibrium exists and optimal. This paper may be extended to evaluate interest-rate and inflation by making some additional simple calculations.

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